A Note on Turbulent Swirling Flows

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Nomenclature

A	= total flux of the angular momentum, Eq. (17)
A^*	= total flux of the angular momentum at any reference station, Eq. (7)
b	= constant in Eq. (7)
B	= constant
C_f	= coefficient of friction
d	= pipe diameter at any radius r
D	= pipe diameter
D_H	= hydraulic diameter
f	= function of r
F	= function of Z
F'	= derivative with respect to Z
K	= constant in Eq. (2)
m_2, m_1	= nondimensional intensity of swirl at a downstream and an upstream station, respectively, Eq. (8)
r_i, r_0	= inner and outer cylinder radius, respectively
r, ϕ, Z	= radial, tangential, and axial coordinates
V, W, U	= radial, tangential, and axial velocities
w', v'	= fluctuating components
ν	= kinematic viscosity
$\epsilon_{r\phi}$	= tangential eddy viscosity

Introduction

THE application of swirling flow, flow with a tangential velocity component in addition to the normal axial component, in the combustion field is wide and varied. Swirling flows are also used to augment heat-transfer coefficients in heat exchangers.[1]

Turbulent swirling flows have been studied by many investigators along with their transport properties.[2-4] An informative review on the turbulent transport properties in such flows has been given by Scott and Rask.[4] The most important among these properties is the eddy diffusivity of momentum, and this forms the subject matter of a number of investigations both experimental and analytical.

It is well known that the eddy diffusivity of momentum varies from point to point in swirling flows and that it is also not isotropic. At any location in the flow, therefore, the diffusivities may be different in different directions. Among these, however, the diffusivity $\epsilon_{r\phi}$ is probably of the greatest significance from a practical point of view. It is this component of diffusion that has been investigated most. It can be determined by hot-wire measurements from the relation:

$$\epsilon_{r\phi}\frac{\partial W}{\partial r} = -\overline{w'v'} \tag{1}$$

Many analytical investigations of this component of diffusion are based upon Prandtl's mixing length theory.

If one examines the swirling flow in a duct, one may observe that the tangential velocity field may be divided into two regions: a central region which approximates a forced vortex (W = constant $\times r$), surrounded by an outer region which approximates a free vortex (W = constant/r).

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Kreith and Sonju[2] and Scott[5] have assumed the eddy viscosity as constant for swirling flow in a duct. Rochinio and Lavan,[3] in a study of swirling flow in a duct, used the relation

$$\epsilon_{r\phi} = K^2 r^2 \left| \left(\frac{\partial W}{\partial r} - \frac{W}{r}\right) \right| \tag{2}$$

between eddy viscosity and shear velocity. They claimed that this model could not be used in the zone with solid body rotation. There they assumed that the eddy viscosity is the same in all directions and proportional to the axial Reynolds number.

Scott and Rask,[4] in a study of turbulent viscosity in swirling flow in an annular duct, have evaluated both the axial and the tangential diffusivities from the conservation equations, using mean values of the velocities and neglecting the turbulent fluctuation terms. They have concluded that the tangential diffusivities vary as in the case of a curved channel. In a curved channel the eddy viscosities near the outer wall are higher than those near the inner wall. This is no doubt because a concave surface promotes instabilities in a flow past it, whereas a convex surface has a stabilizing influence.

In this Note, the equations governing swirling flows are examined, and a simple expression is derived for the diffusivity $\epsilon_{r\phi}$.

Theoretical Analysis

We consider swirling flow in either a cylindrical duct or the annulus between two coaxial cylinders. We make the assumption that the flow approximates well the combination of a forced vortex surrounded by a free vortex. This assumption is supported by experimental results.[2-4,6]

In steady, incompressible turbulent flow that is also axisymmetric, the swirl equation, i.e., the equation of motion in the ϕ direction, simplifies to[2]

$$U\frac{\partial W}{\partial Z} + V\frac{\partial W}{\partial r} + \frac{VW}{r} = (\nu + \epsilon_{r\phi})\left[\frac{\partial^2 W}{\partial Z^2} + \frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} - \frac{W}{r^2}\right] \tag{3}$$

Assuming that the radial velocity V is negligible, Eq. (3) reduces to

$$U\frac{\partial W}{\partial Z} = (\nu + \epsilon_{r\phi})\left[\frac{\partial^2 W}{\partial Z^2} + \frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} - \frac{W}{r^2}\right] \tag{4}$$

However, for both a forced vortex and a free vortex, the terms $[(\partial^2 W/\partial r^2) + (1/r)(\partial W/\partial r) - (W/r^2)]$ add up to zero. Equation (4) therefore, reduces to a simple form

$$U\frac{\partial W}{\partial Z} = (\nu + \epsilon_{r\phi})\frac{\partial^2 W}{\partial Z^2} \tag{5}$$

Kreith and Sonju have suggested that the swirl velocity W varies with Z according to the relation

$$W = f(r)e^{F(Z)} \tag{6}$$

where

$$F(Z) \rightarrow \frac{\text{constant }(1 + \epsilon_{r\phi})Z}{\text{Reynolds no.}}$$

Wolf et al.[7] also have found that the decay of swirl in a pipe flow is exponential. If the total flux of angular momentum is A at the measuring station and that at a reference station is A^*, they have suggested that the swirl decays as

$$\frac{A}{A^*} = \exp\left[-b\left(\frac{Z}{D} - 2\right)\right] \tag{7}$$

Table 1 Experimental data for an annular duct[a]

$\frac{r-r_i}{r_0-r_i}$	0.2		0.4		0.6		0.8	
Z/D_H	W, fps	U, fps	W, fps	U, fps	W, fps	U, fps	W, fps	U, fps
1.7	110.0	97.5	94.0	105.0	77.0	98.6	67.0	82.0
7.0	92.0		87.0		76.0		67.0	
14.8	64.0	86.5	73.0	92.0	67.0	93.5	60.5	88.0
22.2	56.5	94.0	61.0	101.0	57.5	99.5	56.5	92.5
32.7	36.0	91.6	49.0	94.0	53.0	92.5	51.5	88.6

[a] d was calculated from $\frac{r-r_i}{r_0-r_i}$ and Z from Z/D_H for $D_H=3.0$ in.

Table 2 Experimental data for a pipe[7]

Radial position (r/D)	0.2			0.3	0.4	
$Z/D=$	W, fps	U, fps	W, fps	U, fps	W, fps	U, fps
2	534	84.4	562.5	253.1	515.6	342.2
18	482	118.7	440.6	315.1	345.0	358.6
36	412	246.1	319.0	315.1	239.0	344.2
68	243	314.0	178.1	324.8	131.25	340.2

Senoo and Nagata[8] have also found that the swirl decay, characterized by the ratio of the angular momentum flux to the axial momentum flux through a cross section of pipe is exponential in (Z/D). The final expression obtained by them is

$$\log_{10}(m_2/m_1) = -1.10C_f(Z/D) \tag{8}$$

Using the experimental values reported in Refs. 4 and 7, for swirling flow through an annular duct and a pipe, respectively, given in Tables 1 and 2, the exponential dependence of the swirl velocities W on (Z/D) is shown in Figs. 1 and 2. In Fig. 1, W is plotted against the nondimensional distance (Z/d) for a given value of d. Figure 2 is a plot of W against (Z/D), where D is the pipe diameter. It is seen that in those experiments, also, the same kind of exponential decay of W is obtained. We can therefore conclude that the swirl velocity W is exponential in (Z/D).

The variation of W with the radial direction r is simply put down as $f(r)$, as it is seen later that this does not influence the expression for $\epsilon_{r\phi}$.

We shall therefore assume

$$W=f(r)e^{F(Z/D)} \tag{9}$$

where $F(Z/D)$ is linear in Z. It can be seen that this form of W is suggested by relations (6-8).

For the decaying swirl, the exponent $F(Z/D)$ is negative. This, however, does not match the signs of the terms on the two sides of Eq. (5). This difficulty can be avoided by writing Eq. (5) as

$$U\left|\frac{\partial W}{\partial Z}\right| = (\nu+\epsilon_{r\phi})\frac{\partial^2 W}{\partial Z^2} \tag{10}$$

From the relation in Eq. (9), we have

$$\frac{\partial W}{\partial Z} = f(r)F'(Z/D)e^{F(Z/D)} \tag{11}$$

$$\frac{\partial^2 W}{\partial Z^2} = f(r)[F'(Z/D)]^2 e^{F(Z/D)} \tag{12}$$

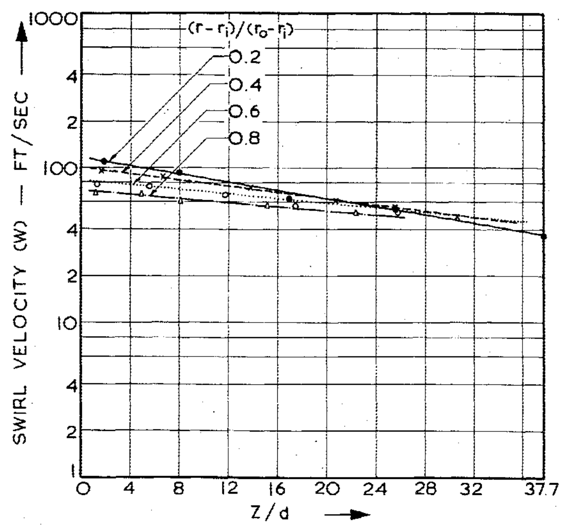

Fig. 1 Exponential variation of the swirl velocity W for swirling flow through an annular duct.[4]

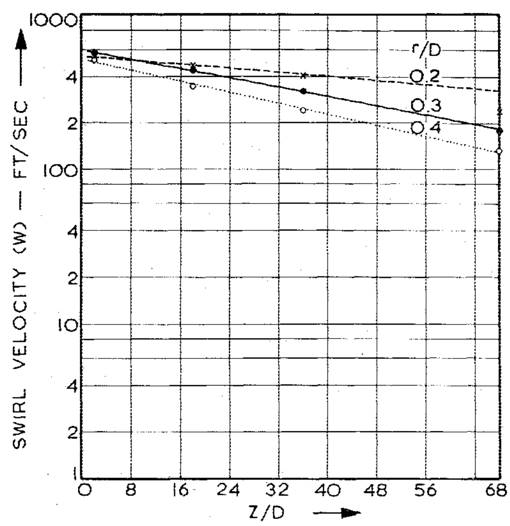

Fig. 2 Exponential variation of the swirl velocity W for swirling flow through a pipe.[7]

Substituting these values in Eq. (5), we have

$$(\nu+\epsilon_{r\phi}) = \frac{U\,|\partial W/\partial Z|}{\partial^2 W/\partial Z^2} = \frac{U\cdot f(r)\cdot F'(Z/D)\cdot e^{F(Z/D)}}{f(r)[F'(Z/D)]^2\cdot e^{F(Z/D)}}$$

$$= \frac{U}{|F'(Z/D)|} \tag{13}$$

If $F(Z/D)$ is of the form $B(Z/D)$, where B is constant, we have from Eq. (13),

$$(\nu+\epsilon_{r\phi}) = (U\cdot D)/B \tag{14}$$

Then, for a given value of D (or D_H, as the case may be)

$$(\nu+\epsilon_{r\phi}) \propto U \tag{15}$$

This relation shows that $\epsilon_{r\phi}$, is no longer constant in a swirling flowfield, but that it varies directly as the axial velocity U. At the wall, where $U=0$, the eddy viscosity vanishes. This expression for $\epsilon_{r\phi}$ is independent of velocity derivatives and it will therefore never attain values of either zero or infinity as may happen when it is expressed in terms of the velocity derivatives.[4]

Equation (14) is simple, and, using this, the eddy viscosity ($\epsilon_{r\phi}$) can be evaluated from measured mean values of the axial velocities, provided that the constant B is evaluated. This constant can be evaluated by plotting the decay of swirl velocities against (Z/D).

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Tunnel Interference Assessment by Boundary Measurements

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Introduction

A NEW approach to assess and correct wind tunnel interference is urgently needed because of the limitation of the classical theory. The classical method[1] requires a definition of the tunnel wall boundary condition and a mathematical model of the test article. The definition of the wall condition is one of the obstacles to the routine application of classical wind tunnel theory to ventilated transonic wall tunnels.[2] Furthermore, the representation of the test model by certain combinations of singularities neglects all viscous effects. Contrary to the classical approach, a new proposed method, which attempts to avoid these difficulties, requires only the measurement of flow variables at a control surface near the tunnel wall inside the test section. The objective of this Note is to present the new approach and the related derivation of the interference flowfield based on measurements at the control surface. Experimental verification of the new concept is also included.

Formulation

A two-dimensional example is chosen to illustrate the formulation of the computation procedure. The Fourier transform technique is applied to the subsonic flow case to obtain the interference flowfield and the flow variables under free-air conditions at the control surface.

The boundary value problem shown in Fig. 1 is formulated to simulate a wind tunnel experiment including measured boundary values (flow variables) at a selected control surface near the tunnel wall, in addition to the regularly measured quantities on the test article. A functional relationship between flow perturbation variables $U_T(x,h)$, $V_T(x,h)$ at the control surface and the model geometry can be obtained by the Fourier transform technique in the transformed plane as follows:

$$\bar V_T(p,h) = i\beta\frac{p}{|p|}\,\bar U_T(p.h)\tanh(|p|\beta h) + \bar F(p)\cdot \mathrm{sech}(p\beta h) \tag{1}$$

where

$$\bar F(p) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} F_x(x)e^{ipx}\mathrm{d}x$$

and p is the Fourier transform parameter. The model geometry F should be interpreted as a "potential equivalent" airfoil profile including viscous effects. It is reasonable to assume that the same equivalent profile exists in a free-air flow condition. The solution for free-air flow is readily obtained in the transformed plane

$$\bar U_{\text{free air}} = i\frac{p}{\beta|p|}\bar F(p)e^{-|p|\beta h} \tag{2a}$$

$$\bar V_{\text{free air}} = \bar F(p)e^{-|p|\beta h} \tag{2b}$$

Combining Eqs. (1) and (2) to eliminate the model geometry F one obtains the free-air flowfield directly related to the measured tunnel flow perturbation variables U_T, V_T at the control surface without explicit terms of the model geometry.

$$\bar U_{\text{free air}} = e^{-|p|\beta h}\left(\bar U_T \sinh|p|\beta h + \bar V_T\frac{ip}{\beta|p|}\cosh p\beta h\right) \tag{3a}$$

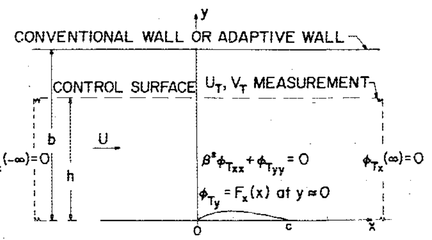

Fig. 1 Boundary-value problem of tunnel simulation with boundary measurements.

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